1) Find a unit normal vector to the surface at the given point.

a) $x^2y^3 - y^2z + 2xz^3 = 4$, $(-1,1,-1)$

b) $\sin(x-y) - z = 2$, $\left(\frac{\pi}{3}, \frac{\pi}{6}, -\frac{3}{2}\right)$

a) $\boxed{-\frac{4}{3\sqrt{10}}\mathbf{i} + \frac{5}{3\sqrt{10}}\mathbf{j} - \frac{7}{3\sqrt{10}}\mathbf{k}}$

b) $\boxed{\frac{\sqrt{30}}{10}\mathbf{i} - \frac{\sqrt{30}}{10}\mathbf{j} - \frac{2\sqrt{5}}{10}\mathbf{k}}$

2) Find an equation of the tangent plane to the surface at the given point.

a) $z = x^2 + y^2 + 3$, $(2,1,8)$

b) $x = y(2z-3)$, $(4,4,2)$

a) $\boxed{4x + 2y - z = 2}$

b) $\boxed{-x + y + 8z = 16}$

3) Find an equation of the tangent plane and find symmetric equations of the normal line to the surface at the given point.

a) $xyz = 10$, $(1,2,5)$

b) $y \ln xz^2 = 2$, $(e,2,1)$

a) $\boxed{\text{Plane: } 10x + 5y + 2z = 30, \text{ Line: } \frac{x-1}{10} = \frac{y-2}{5} = \frac{z-5}{2}}$

b) $\boxed{\text{Plane: } \frac{2}{e}x + y + 4z = 8, \text{ Line: } \frac{x-e}{\frac{2}{e}} = \frac{y-2}{1} = \frac{z-1}{4}}$

4) Given the surface $x^2 + y^2 + z^2 = 14$ and the surface $x - y - z = 0$ find the following:

a) Symmetric equations of the tangent line to the curve of intersection of the surfaces at the point $(3,1,2)$.

b) The cosine of the angle between the gradient vectors at the point $(3,1,2)$.

c) Determine whether or not the surfaces are orthogonal at the point of intersection $(3,1,2)$.

a) $\boxed{\text{Line: } \frac{x-3}{1} = \frac{y-1}{5} = \frac{z-2}{-4}}$

b) $\boxed{\cos\theta = 0}$

c) $\boxed{\text{Orthogonal}}$

5) Find the angle of inclination θ of the tangent plane to the surface at the given point.

a) $3x^2 + 2y^2 - z = 15$, $(2,2,5)$

b) $x^2 + y^2 = 5$, $(2,1,3)$

a) $\boxed{\theta \approx 86.03^\circ}$

b) $\boxed{\theta = 0^\circ}$

6) Find the point(s) on the surface at which the tangent plane is horizontal.

a) $z = 3 - x^2 - y^2 + 6y$

b) $z = 5xy$

a) $\boxed{(0,3,12)}$

b) $\boxed{(0,0,0)}$

7) Show that the surfaces $x^2 + 2y^2 + 3z^2 = 3$ and $x^2 + y^2 + z^2 + 6x - 10y + 14 = 0$ are tangent to each other at the point $(-1,1,0)$ by showing that the surfaces have the same tangent plane at this point.

$$\boxed{-x + 2y = 3}$$

8) Show that the surfaces $z = 2xy^2$ and $8x^2 - 5y^2 - 8z = -13$ intersect at the point $(1,1,2)$, and show that the surfaces have perpendicular tangent planes at this point.

$$\boxed{F(1,1,2) = G(1,1,2) = 0,\ \nabla F \cdot \nabla G = 0}$$

9) Find the point on the hyperboloid $x^2 + 4y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $x + 4y - z = 0$.

$$\boxed{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \text{ and } \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)}$$

10) Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(-1, 1, 2)$.

$$\boxed{x = -1 - 10t,\ y = 1 - 16t,\ z = 2 - 12t}$$